Simulation of never changed opinions in Sznajd consensus model using multi-spin coding

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Abstract: The density of never changed opinions during the Sznajd consensus-finding process decays with time t as $1/t^\theta$. We find $\theta \simeq 3/8$ for a chain, compatible with the exact Ising result of Derrida et al. In higher dimensions, however, the exponent differs from the Ising θ. With simultaneous updating of sublattices instead of the usual random sequential updating, the number of persistent opinions decays roughly exponentially. Some of the simulations used multi-spin coding.

Keywords: Persistence exponent, single-bit handling, dimensionality dependence.

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In the Ising and Potts models, the persistent spins [1] are those which from the beginning of a (zero-temperature) Monte Carlo simulation have never been flipped. In the thermodynamic limit, their number decreases with time t asymptotically as $1/t^\theta$ with $\theta = 3/8$ exactly on the Ising chain [2], while higher dimensions were investigated numerically [3] giving $\theta \simeq 0.2$ on the square lattice. Also the more general Potts model was investigated [1,2,3]. Now we simulate the analogous number in a d-dimensional Sznajd model of consensus-finding [4] with up to 49 million sites and $1 \le d \le 4$, for the case of just two possible opinions, the equivalent of spin 1/2 Ising sites.

In this Sznajd model (see [5] for a review) two opposing opinions are initially distributed randomly with equal probability over the L^d "people" of a hypercubic lattice. Then, each randomly selected pair of nearest neighbours convinces its $4d-2$ nearest neighbours of the pair opinion if the pair shares the same opinion; otherwise, the neighbour opinions are not affected. One time step means that on average every lattice site is selected once as the

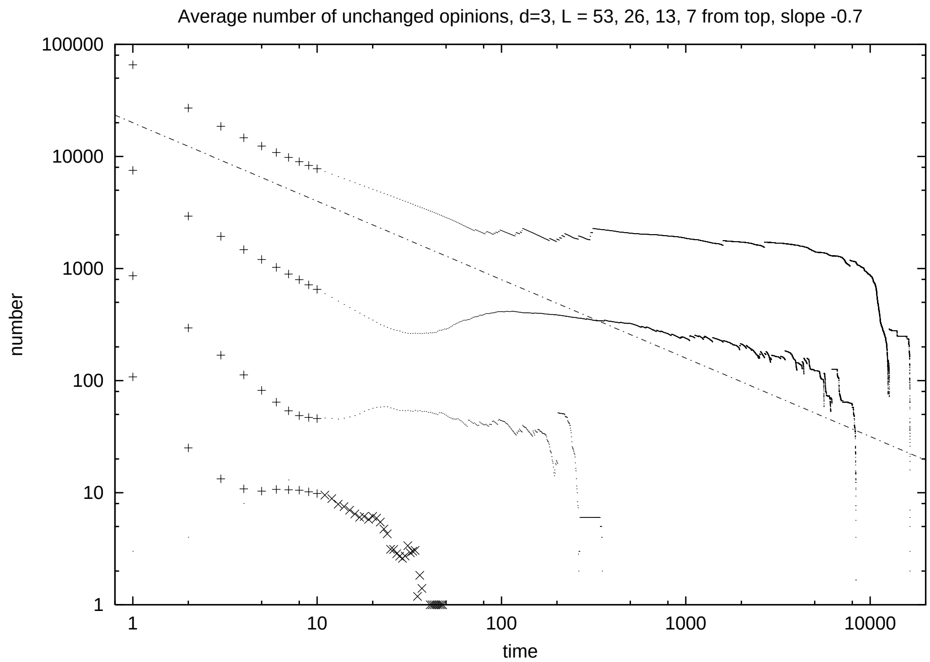

Figure 1: Log-log plot of the number of persistent people, $P(t) - P(\infty)$, versus time, for one, 10^3, 10^3, 10^4 simple cubic lattices of size $L \times L \times L$ with $L = 53$, 26, 13, 7. Only the intermediate times before the plateau and final decay to zero are used to estimate the exponent $\theta \simeq 0.7$, indicated here by the straight line.

first member of the pair. (We will mention below the different results if this random sequential updating is replaced by simultaneous updating.) The Sznajd model is one of several recent consensus-finding models [6] and follows a long tradition of social studies using computer simulation and/or statistical physics [7]. If we wait sufficiently long for large systems, always a consensus is found: Everybody has the same opinion and the whole system has reached a fixed point.

Alternatively, independently of social interpretations, this model can be understood as a variant of the traditional kinetic Ising model: instead of a central site being influenced by its neighbourhood, the neighbourhood itself is updated according to the states of the central spins.

We check for the number $P(t)$ of "persistent" sites who have not yet

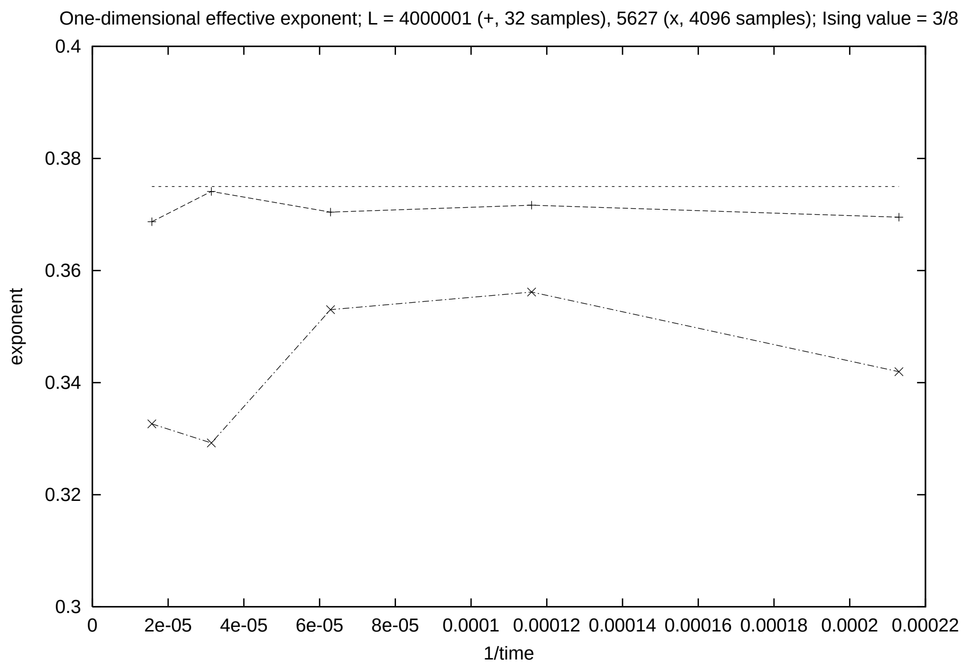

Figure 2: Effective exponents in one dimension, approaching perhaps the Ising value 3/8 (horizontal line) for long times and large lattices.

changed their spins in this Sznajd consensus process. (All our sites are equivalent, in contrast to Schneider's modification [8] where some sites are initially selected as permanent opponents.) We find that usually a consensus is found before everybody had changed opinion; i.e. $P(\infty) > 0$. Thus the exponent θ has to be determined from intermediate times where $P(0) \gg P(t) \gg P(\infty)$, or from $P(t) - P(\infty)$. Fig.1 shows that this latter quantity has a complicated behaviour, and again only intermediate times are used to find θ. In one dimension, $P(\infty)$ is relatively small and the resulting systematic deviations disturb less.

Figs.2 to 4 show for $d = 1$, 2 and 3 the effective exponents θ analyzed by least square fits over five suitable time intervals $t_n < t < t_{n+1}$ with $t_{n+1} = 2t_n$. For $d > 1$, only the times until the first of the (typically 1000) samples reached a consensus were used and averaged over. We conclude that $\theta \simeq 3/8$ in one dimension, 0.5 in two, and the same or somewhat higher in three dimensions. Fig.5 shows that four dimensions is difficult to analyze

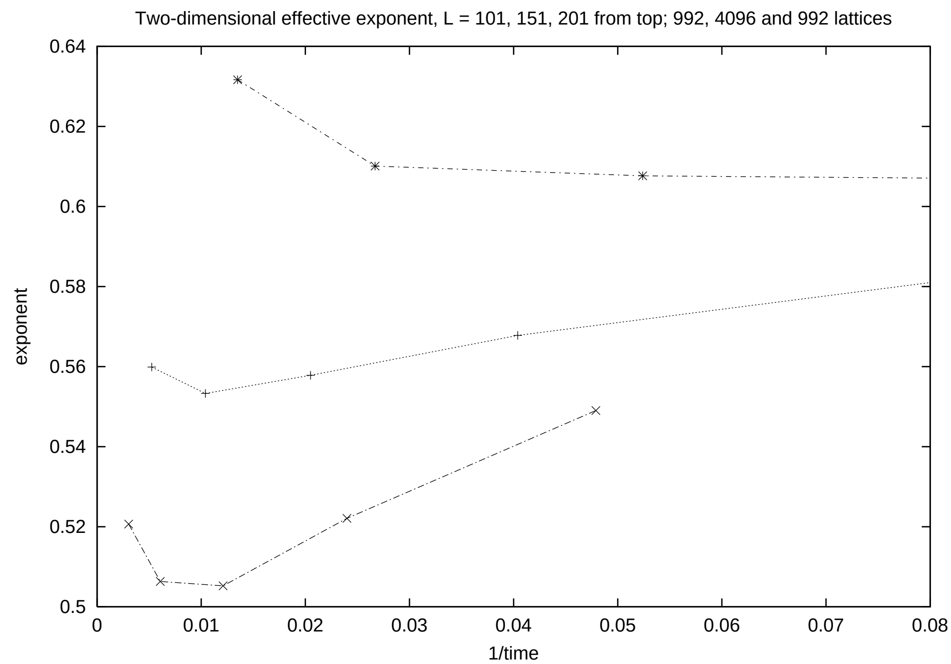

Figure 3: As Fig.2, but for square lattices, using intermediate times only, before any final consensus was found.

though maybe $\theta \simeq 0.9$. Our one-dimensional estimate is compatible with the Ising value 3/8, but for higher dimensions our $\theta(d)$ goes up while the Ising $\theta(d)$ went down for increasing d.

We also speeded up the simulations by storing 32 or 64 sites (belonging to 32 or 64 different samples) in each computer word, using single-bit handling [9] known for Ising models as multi-spin coding. The random selection of neighbour pairs was the same for all 32 or 64 samples. The C program is available from PMCO, the Fortran program from DS.

If during one time step all sites are updated simultaneously, with frustrated sites not changing their opinion, then no consensus is found [10]. (Frustrated are those sites which simultaneously are convinced to different opinions by different neighbour pairs.) This frustration can be avoided by dividing the lattice into sublattices, such that no sites within one sublattice can influence each other directly; we divided our lattices such that the distances between sites belonging to the same sublattice are at least five lattice constants. (For

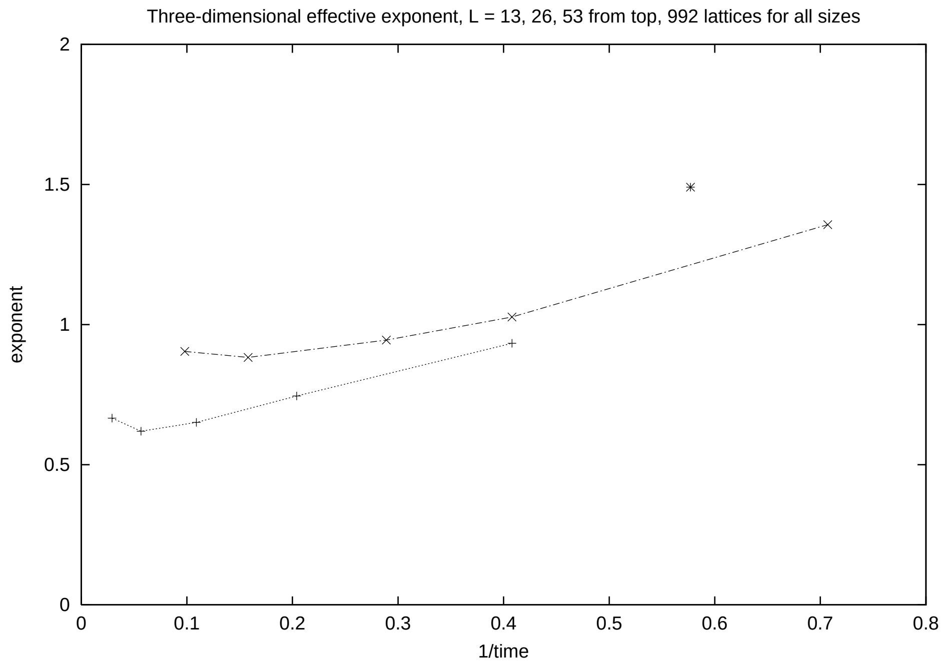

Figure 4: As Fig.3 but for simple cubic lattices.

nearest-neighbour Ising models, the two sublattices of a chessboard suffice on the square lattice, while we used 25 interpenetrating sublattices for the square Sznajd model.) With this simultaneous updating of sublattices, frustration is avoided, a consensus is always found, but $P(t)$ no longer decays as a power law, Fig.6: criticality seems lost. Also, this version no longer shows the phase transition of the usual square Sznajd model, when the ratio of the fractions of the two initial opinions is varied away from unity. Thus, the simultaneous updating is not merely a possible acceleration of the dynamic process. Also the correlations between spins behave differently, being affected by the simultaneous updating of spins far apart from each other.

In summary, the Sznajd model is Ising-like in one dimension but not in higher dimensions for the persistence exponent θ.

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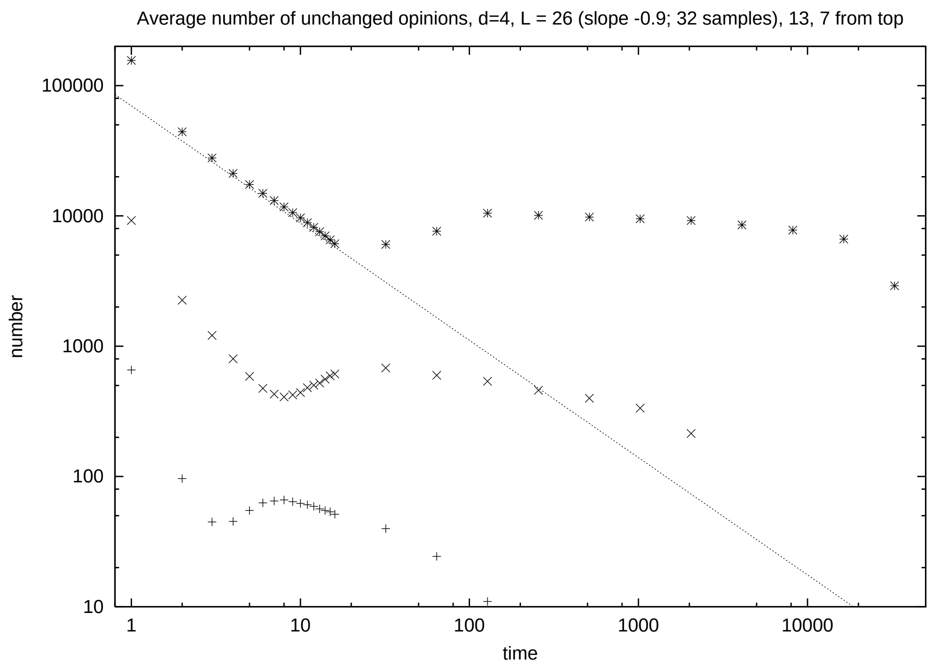

Figure 5: Log-log plot of $P(t) - P(\infty)$ for hypercubic lattices in four dimensions averaged over 10^3 samples (32 samples only for $L = 26$).

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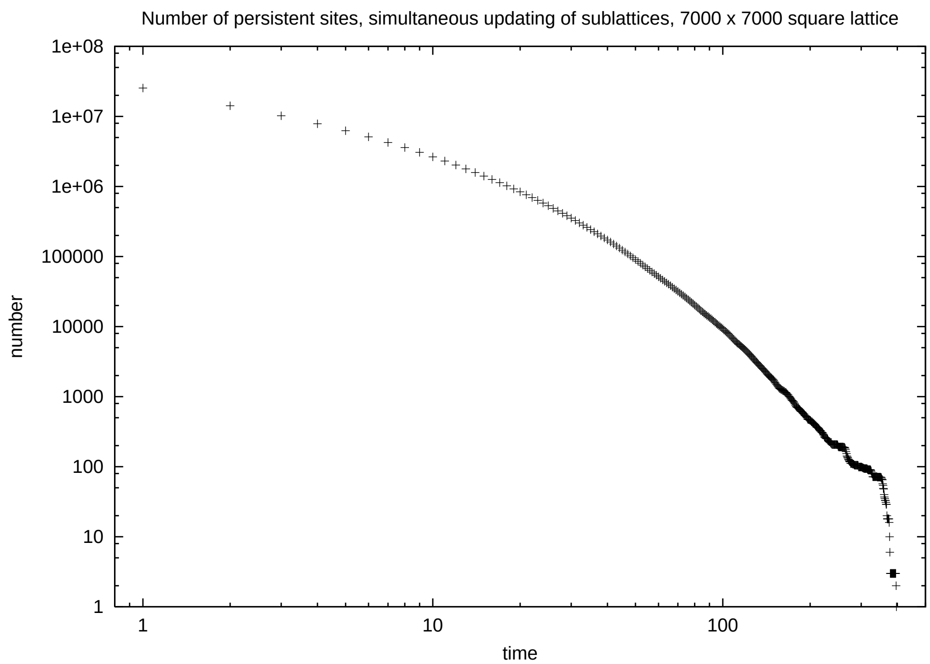

Figure 6: Log-log plot of $P(t)$ for 7000×7000 square lattice with unfrustrated simultaneous updating of sublattices.

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